

1. For which values of n do the following graphs have an Euler circuit? Explain why.

- (a) K_n (b) C_n (c) W_n (d) Q_n

A connected multigraph (or graph) has an Euler circuit iff each of its vertices has even degree.

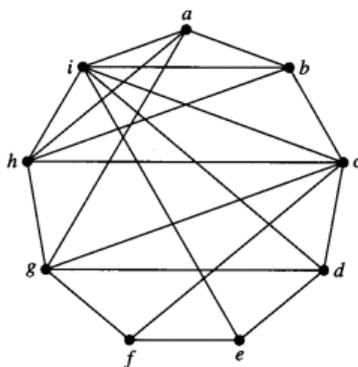
(a) Every vertex in K_n has degree $n-1$. K_n has an Euler circuit if n is odd.

(b) Every vertex in C_n has degree 2. C_n has an Euler circuit for every n .

(c) Every vertex except the center of W_n has degree three. W_n has an Euler circuit for no n .

(d) Every vertex in Q_n has degree n . Q_n has an Euler circuit if n is even.

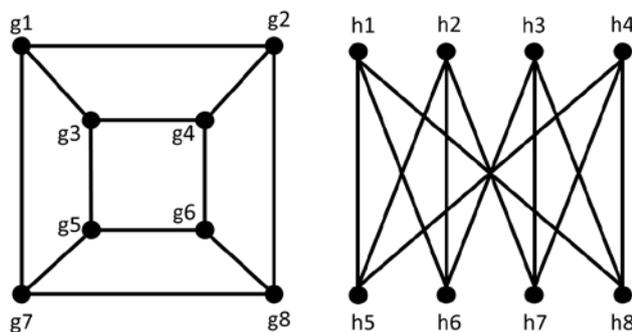
2. Find the chromatic number of the given graph.



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3. Check whether the following pairs of graphs are isomorphic. If they are, find the bijection between the vertex sets of G and H that $f = V(G) \rightarrow V(H)$

(a)



graph G

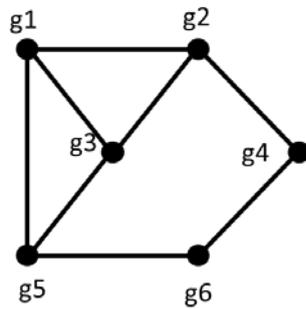
graph H

Isomorphism.

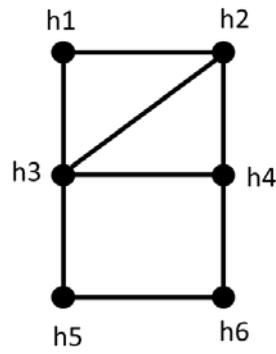
$f(g_1)=h_1$; $f(g_2)=h_5$; $f(g_3)=h_6$; $f(g_4)=h_2$;

$f(g_5)=h_3$; $f(g_6)=h_7$; $f(g_7)=h_8$; $f(g_8)=h_4$;

(b)



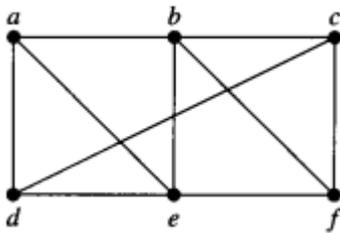
graph G



graph H

Not isomorphism.

4. Find the number of path between c and d of length 4.



Ans:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 3 & 1 & 2 & 1 & 2 & 2 \\ 1 & 4 & 1 & 3 & 2 & 2 \\ 2 & 1 & 3 & 0 & 3 & 1 \\ 1 & 3 & 0 & 3 & 1 & 2 \\ 2 & 2 & 3 & 1 & 4 & 1 \\ 2 & 2 & 1 & 2 & 1 & 3 \end{bmatrix} \quad A^3 = \begin{bmatrix} 4 & 9 & 4 & 7 & 7 & 5 \\ 9 & 6 & 9 & 4 & 10 & 7 \\ 4 & 9 & 2 & 8 & 4 & 7 \\ 7 & 4 & 8 & 2 & 9 & 4 \\ 7 & 10 & 4 & 9 & 6 & 9 \\ 5 & 7 & 7 & 4 & 9 & 4 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 23 & 20 & 21 & 15 & 25 & 20 \\ 20 & 35 & 17 & 28 & 26 & 25 \\ 21 & 17 & 24 & 10 & 28 & 15 \\ 15 & 28 & 10 & 24 & 17 & 21 \\ 25 & 26 & 28 & 17 & 35 & 20 \\ 20 & 25 & 15 & 21 & 20 & 23 \end{bmatrix}$$

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